Digital Image Release Form

I [print name] MICHAEL J McMAHON give permission to the University of Detroit Mercy to include a digital image of my thesis/dissertation titled: DYNAMIC TORSION OF THIN-WALLED TUBES (TZ-M2275) on its Library/IDS website.

By means of my signature, the following is understood and meets with my agreement:

- Copyright to the above named thesis/dissertation will be retained by me;
- Copyright to the digital database will be retained by the University of Detroit Mercy;
- Library/IDS will make the digitized image available to the public by means of its website and will provide and maintain necessary service space;
- If not already in digital form, Library/IDS will be responsible for scanning the thesis/dissertation and preparing the initial searchable text by means of optical character recognition (OCR) software;
- Library/IDS will proofread and correct the OCR text.

Signed Michael J McMahon Date 7-7-11

Signed Margaret E Auer Date 7/11/11
Margaret E. Auer, Dean
University Libraries/Instructional Design Studio

THE UNIVERSITY OF DETROIT

DYNAMIC TORSION OF THIN-WALLED TUBES

A THESIS

SUBMITTED TO THE FACULTY

OF THE COLLEGE OF ENGINEERING

IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF

BACHELOR

OF

MECHANICAL ENGINEERING

BY

MICHAEL J. MCMAHON

DETROIT, MICHIGAN

MAY, 1964

BURSAR Mary M. [illegible]

ADVISOR Thomas Ufaust Ph.D.

PREFACE AND ACKNOWLEDGEMENTS

Many analyses of the elastic stability of thin-walled cylinders subjected to loads of compression, tension, and torsion have been made. In recent years due to the wider use of thin-walled cylinders in aircraft structures, the stability of thin-walled cylinders in torsion has been more completely investigated.

Although considerable work has been done both analytically and experimentally on thin-walled cylinders subjected to torsional loads, it has been limited to full round, seamless cylinders subjected to a static torsion. It is the purpose of this thesis to present the problem of predicting the dynamic torque capabilities and the characteristic deformation of thin-walled tubes which are full round and welded. This was done by means of stress analysis and concurred by laboratory experiment.

The author wishes to thank Dr. Thomas Manos for his helpful advice, Charles DeLand for his assistance as my thesis lab partner, and the machine shop employees for their help in preparing the apparatus.

TABLE OF CONTENTS

SUMMARY

The following investigation showed that when thin-walled tubes fail in elastic instability from a dynamic torsion; there will be a characteristic deformation in this failure. The characteristic deformation is circumferential buckling waves along the length of the tube.

The important result found is that the distance from the point of torsional impact to the location of the buckling is dependent only on the speed at which the torsion is applied and not on the load applied. As the speed of torsion increases, the distance from the point of torsional impact to the location of the buckling decreases.

CHAPTER I

INTRODUCTION

When considering the design of today's lightweight structures and their use under impact loads, the investigation of the dynamic torsional characteristics related to the stability of thin-walled tubes becomes important. In aircraft structures where weight and strength are of equal importance, it is necessary to be able to accurately predict the capabilities of every structural member so that they may be safely loaded to their full capacity. When a dynamic torque resulting from a sudden acceleration or deceleration caused by such things as the take-off, landing, air pockets, and changing wind velocity is applied to the thin-walled structures, the members are subject to a loading condition which affects the stability of the members at various locations determined by the speed at which the load is applied.

This phenomenon can be better realized by the observation of a damaged vehicle after collision. An inspection of a damaged fender will reveal many ripples located close to the place of impact. It has been observed that if the same fender was crushed by a slowly applied equivalent load the ripples would probably be just as significant,but they would be located on the fender further away from the place of impact.

It has been found that there has been little if any research conducted on dynamic torsional elastic stability characteristics. This field is extensive and relatively uninvestigated; however this thesis will be limited to the analysis of the characteristic locations of deformation caused by the buckling of thin-walled tubes subjected to dynamic torque.

The method of solving this problem will be to correlate the torsional theory of thin-walled tubes with that of the buckling characteristics of plates. Therefore, the theory of torsion and elastic stability will be developed. The solution will also include various theories on the behavior of metals under impulsive loads.

Previous investigations show that structures such as thin-walled tubes fail in most cases not due to high stresses surpassing the strength of the material, but due to insufficiant elastic stability of the slender structures. The failure of the thin-walled tubes used in this investigation due to elastic instability was assured by their design dimensions despite the increase in the yield strength of the metal.

The analysis of the failure characteristics of tubing under a dynamic torque is not analyzed by the standard torsional strength equations, but instead by resolving the problem into one of local buckling of a curved plate subjected to a compression load. An analysis that would allow the exact relationship of critical torque and buckling characteristics to time rate of torque application is beyond the scope of this thesis. However, an attempt will be made to predict this only in so far as setting up a solution.

The assumptions and basis of this preliminary solution are concurred by laboratory testing of thin walled tubes. The laboratory results very clearly display the existance of the phenomenon which excited this investigation.

CHAPTER II

THEORY OF TORSION

An understanding of the nature of torque and the distribution of the forces in a thin-walled tube as a result of the applied torque is important in the stress analysis of this investigation.

A structure with a twisting couple on each of its ends acting in opposite directions is subjected to a torque and its magnitude is expressed in terms of a force and distance. This twisting causes a tendency of one cross section to rotate about a longitudinal axis with respect to an adjacent cross section, and a tendency for any differential area in one cross section to slide along the corresponding differential area in the adjacent cross section, thus developing shearing stress on all cross sections of the member between the couples. These stresses are called torsional shearing stresses. Figure 1 pictures this stress distribution in the plane of the cross-section and also the complementary shearing stresses in an axial plane.

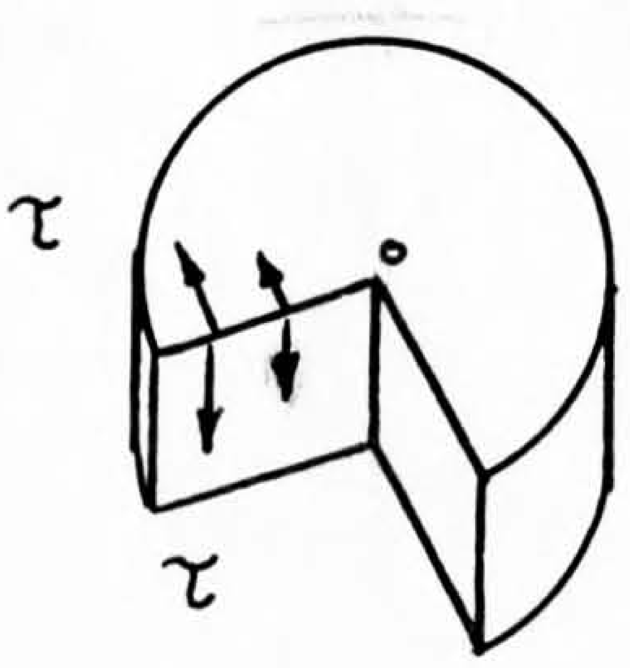


Fig. 1

In the derivation of the basic torsional stress, the following assumptions must be made.

1. That the member is subjected to the action of pure torque. This means there is no bending, tension or compression to exist in the member.

2. The shearing stress on a right cross section is directly proportional to the distance to the center of the member. This implies that the member be circular and perfectly isotropic.

3. The bar has no residual or built-in stresses.

4. The sections under consideration are remote from a change in diameter and from the point of application of load.

The last three assumptions can only be approximated in actual practice.

The product of the torsional shear stress, the area upon which it acts, and the distance to the neutral axis of the member produces an internal resisting torque or stated mathematically, S. Timoshenko and D.H. Young:

$$\tau \rho \, dA = dT$$

The total resisting torque T about the axis of the shaft is the summation, taken over the entire cross-sectional area, of these moments of the individual elements; that is $\Sigma \tau \rho \, dA = T$.

S. Timoshenko and D.H. Young, Strength of Materials, D. Van Nostrand Co., Princeton, New Jersey, 1962. p. 72

But by analytical stress proportion $\tau = \tau_{max.} \rho / r$ where the maximum stress occurs in the outer surface of the member, where $\rho = r$.

In integral form $T = \int \tau_{max.} \frac{\rho}{r} \rho \, dA$

or $T = \frac{\tau_{max.}}{r} \int \rho^2 \, dA$

where $J = \int \rho^2 \, dA$ is defined as the polar moment of inertia of the circular cross section.

Therefore, $T = \frac{\tau_{max.}}{r} J$

Since stress is proportional to strain according to Hooke's Law and strain can be measured directly, stress can be indirectly measured. The angular rotation due to torsional shearing stress is called shearing strain. Shearing strain is defined by the equation $\gamma = r \frac{d\phi}{dx}$ where $\frac{d\phi}{dx}$ is the angle of twist per unit length of the member and will be called θ. Therefore the total angle of twist γ_{TOT} is the product of θ and the total length of the member. $\gamma_{TOT} = \theta \ell$

By substituting these values of shearing stress and strain into the basic deflection equation $\Delta = F\ell / AE$ then the general torsional shearing strain equation is derived, that is $\theta = T\ell / GJ$.

The relationship between pure torsional shear and tension and compression must be understood in order to analyze the causes of buckling in a thin-walled tube. If a shearing stress occurs on a plane at a given point in a stressed body, there must exist a shearing stress of equal magnitude at that point on the second plane at right angles to the first plane as shown in Figure 2. If only shearing stress occurs on two such planes, the state of the stress at the point is said to be "pure shear." This state of stress is shown in Figure 2.

From elementary strength of materials it is known that there are principal stresses accompanying pure shear. If at a point in a body the state of pure shear exists, there also exists tensile and compressive forces which reach their maximum value on the 45° plane as shown in Figure 2b and 2c.

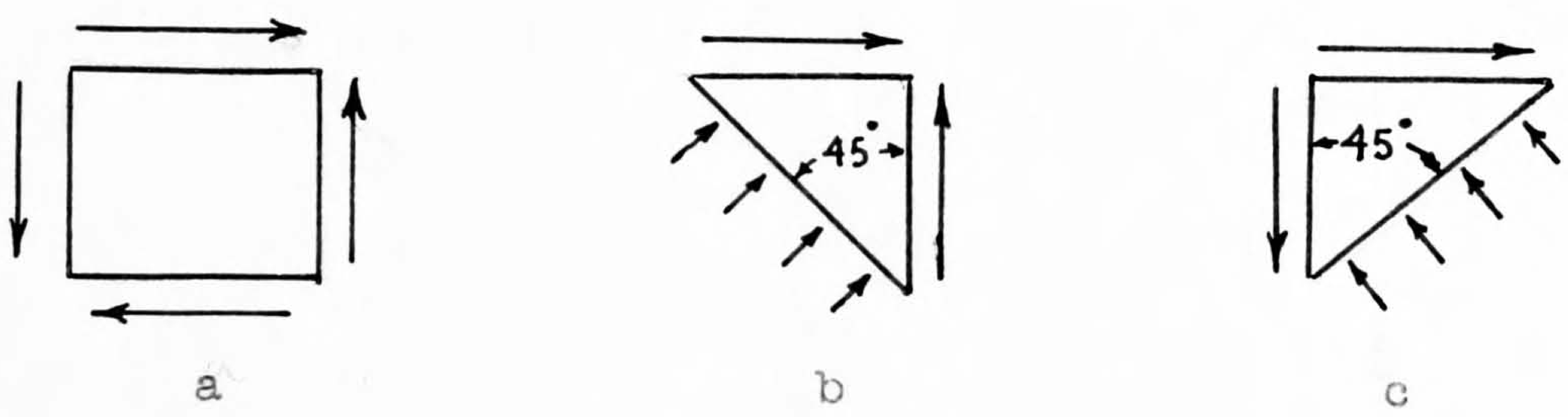
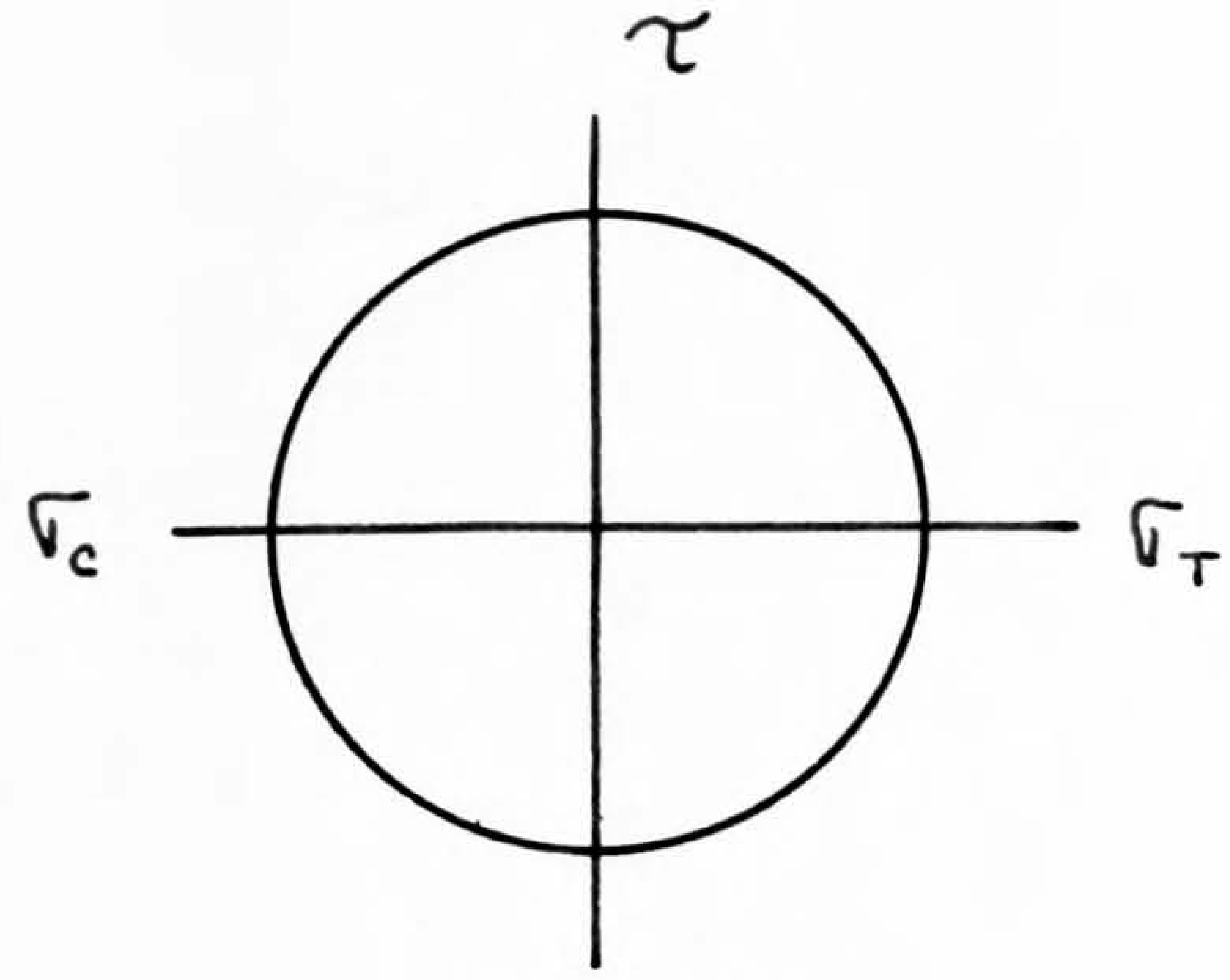


Fig. 2

Thus when a brittle material such as cast iron, which is relatively weak in tension, is subjected to pure shear, the member fails on the plane of maximum tensile stress. These tensile and compressive stresses are equal in magnitude to the shearing stress as shown by a Mohr's Circle diagram of pure torsion.



τ = Torsional Stress

σ_T = Tension Stress

σ_c = Compression Stress

Fig. 3

It is these stresses that cause the critical buckling stress of a thin-walled tube, that is the compressive forces set up by pure shear will buckle the tube by simple compression. This means that thin-walled members subjected to torsion can fail by localized compression or buckling as well as by torsional shear. These induced compressive forces will tend to cause buckling, and this tendency is increased by the fact that the strip is initially bowed due to the curvature of the tube wall. This tendency for buckling is dependent upon the relative thickness, length, and diameter of the cross section and the modulus of elasticity of the material.

<u>Torsional Characteristics of Full Round Tubing</u>. The equation $\tau = \frac{Tr}{J}$ applies to circular tubes whether the walls are thin or thick. After substituting the value of the polar moment of inertia into this equation and simplifying by the approximation that the stress developed in the thin wall is constant, then $\tau = T/2\pi r^2 t$

The derivation for the relationship between the torque and the stress in a thin-walled torsion member was obtained by Glenn Murphy even though the outline of the member is not circular and the wall thickness varies. He uses the term "shear flow" which is defined as the product of the average shearing stress on a transverse section at a point and the thickness of the wall at that point. He states that the shearing stress at any point in the cross section is

$$\text{STRESS} = T/2At$$

"A" is the area enclosed by the centerline of the wall of the tube. "t" is the thickness of the wall at the point considered.

Glenn Murphy, <u>Mechanics of Materials</u>, Irwin-Farnham Publishing Company, Chicago, Illinois, 1948. p. 97

The torsional strain of thin-walled tubing should also be noted. This equation also is only applicable if the thickness of the tube is small compared to the diameter. It is $\phi = \frac{Tl}{2\pi r^3 t G}$ where ϕ is expressed in radians.

CHAPTER III

ELASTIC STABILITY THEORY

An example will best illustrate the theory of elastic stability. This is the example of elastic buckling of compression members where its length is much greater than its radius of gyration with respect to the centroidal axis about which it tends to buckle. This is known as column buckling.

The compression member is a long slender column of length "L" which is built-in at its lower end and subjected to a centrally applied compressive load at its upper end. This column is assumed to be perfectly straight and of uniform cross-section. Also the material is assumed to be homogeneous and behave elastically.

As the load is increased from zero, the column shortens in accordance with Hooke's law and remains in equilibrium. Experience shows that when the vertical load "P" is small, such a compressed column is laterally stable. That is, if the upper end is pushed slightly to one side by a lateral force, the column will return to its straight form as soon as this lateral force is removed. However, as "P" is gradually increased, it is observed that at a certain value of this load, the straight form of

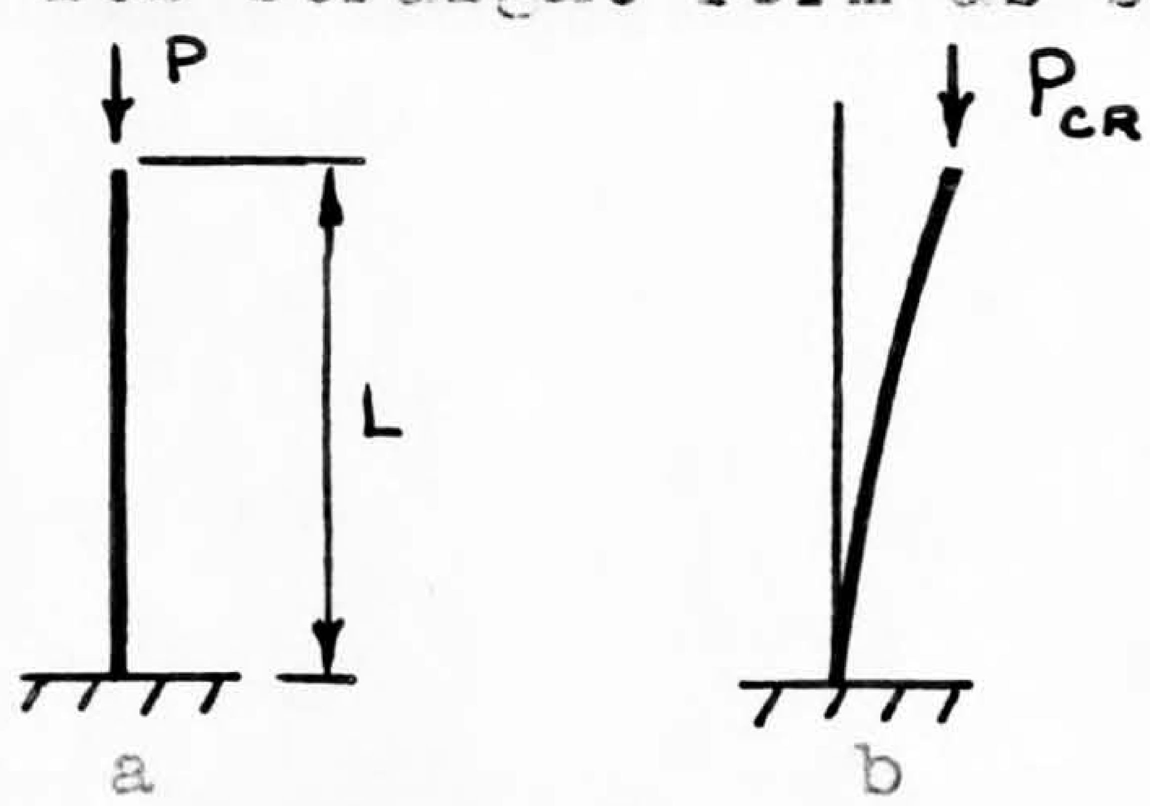


Fig. 4

equilibrium becomes unstable and the column, if pushed to one side, stays there even after the lateral force is removed. This instability phenomenon is called lateral buckling and the value of the load at which it occurs is called the critical load, denoted by P_{cr}.

To find the load P_{cr} which will cause buckling, Timoshenko considered the column in the slightly bent configuration shown in Fig. 4b and calculated the magnitude of the vertical load necessary to hold it there. This load was calculated to be:

$$P_{cr} = \frac{\pi^2 E I}{4 L^2}$$

where E is modulus of elasticity, I is least moment of inertia, L is length of the column.

S. Timoshenko and D.H. Young, Strength of Materials, D. Van Nostrand Co., Princeton, New Jersey, 1962. p. 268.

S. Timoshenko, Theory of Elastic Stability, McGraw-Hill Book Co., New York, 1936. pp. 64 and 66.

He used the differential equation of the deflection curve in the solution. Experiments show that under the action of a load greater than the critical value, a thin member will always buckle sideways.

Now that the elementary case of elastic instability has been discussed, the buckling of thin plates will be investigated for it is this case that includes torsional buckling. In order to calculate the critical values of forces applied axially in the middle plane of a plate at which the flat form of equilibrium becomes unstable, and the plate begins to buckle, the same method as used in column buckling was used by S. Timoshenko.

Professor Timoshenko uses equations of equilibrium for a flat plate in deriving analytically the critical loads. These equations are greatly simplified by L. H. Donnell and referred to in this paper.

L.H. Donnell, Stability of Thin-Walled Tubes under Torsion, National Advisory Committee for Aeronautics, Report No. 479, 1934, p. 105.

The derivations are complex and lengthy, and they will later be discussed only as far as the scope of this thesis permits. To investigate the problem of elastic stability of thin-walled tubes under dynamic torsion by the exact solutions would be more in line with a graduate thesis.

CHAPTER IV

BEHAVIOR OF METALS UNDER IMPULSIVE LOADS

The property of cohesion due to metallic bonds is the main reason of resistance to distortion in metals. The metallic bond which is influential in holding atoms together may be considered to be an attraction between the positive "cores" and the negative unattached electrons of the atoms. The atoms in a metal are pictured as being held in a fixed position by forces, some being attractive and the others being repulsive. If there are only a few valence(outer-shell) electrons within an atom, they may be removed relatively easy, while the balance of the electrons are held firmly to the nucleus. This in effect, forms a structure of positive ions and "free" electrons. The ion "cores" consist of the nucleus and the non-valent electrons.

Because the valence electrons are free to move about within the metal structure, they form what is described as an electron "cloud" or "gas". The positive ions and the negative electron "cloud" provide attractive forces. These forces bond the metal atoms together by an attraction between the nuclei of two adjacent atoms; this attraction is similar to gravity.

In the majority of engineering materials the atoms or groups of atoms are arranged in some regular, repititious pattern which is called a crystal. The strength of a crystalline structure depends upon the cohesion between atoms in the space lattice structure with individual crystalline grains, an upon adhesion between crystalline grains at the grain boundaries.

A grain is a single crystal which usually does not have a regular, external crystalline shape. The shape of a grain in a solid is usually controlled by the presence of surrounding grains. However, at the grain boundary between two adjacent grains there is a transition zone which is not directly aligned with either grain. Grain boundaries affect the mechanical properties of metals. Slip is interrupted by a grain boundary because the plane on which the dislocation moves is terminated. Materials containing many grain boundaries are stronger than identical materials with few boundaries.

If a material is viewed under a microscope after it has been loaded above its elastic limit, fine lines known as slip bands can be seen across the faces of a number of crystalline grains of the overstressed metal as shown in Figure 5 . These bands mark the places where sliding has occurred between thin plates of metal within a crystalline grain. This sliding takes place

M. McMahon, F.P. Miller, J. Wenzler; Effects of Varying Strain Rates on the Dynamic Yield Point; M.E. Lab, Univ. of Detroit, Detroit, Michigan, 1963

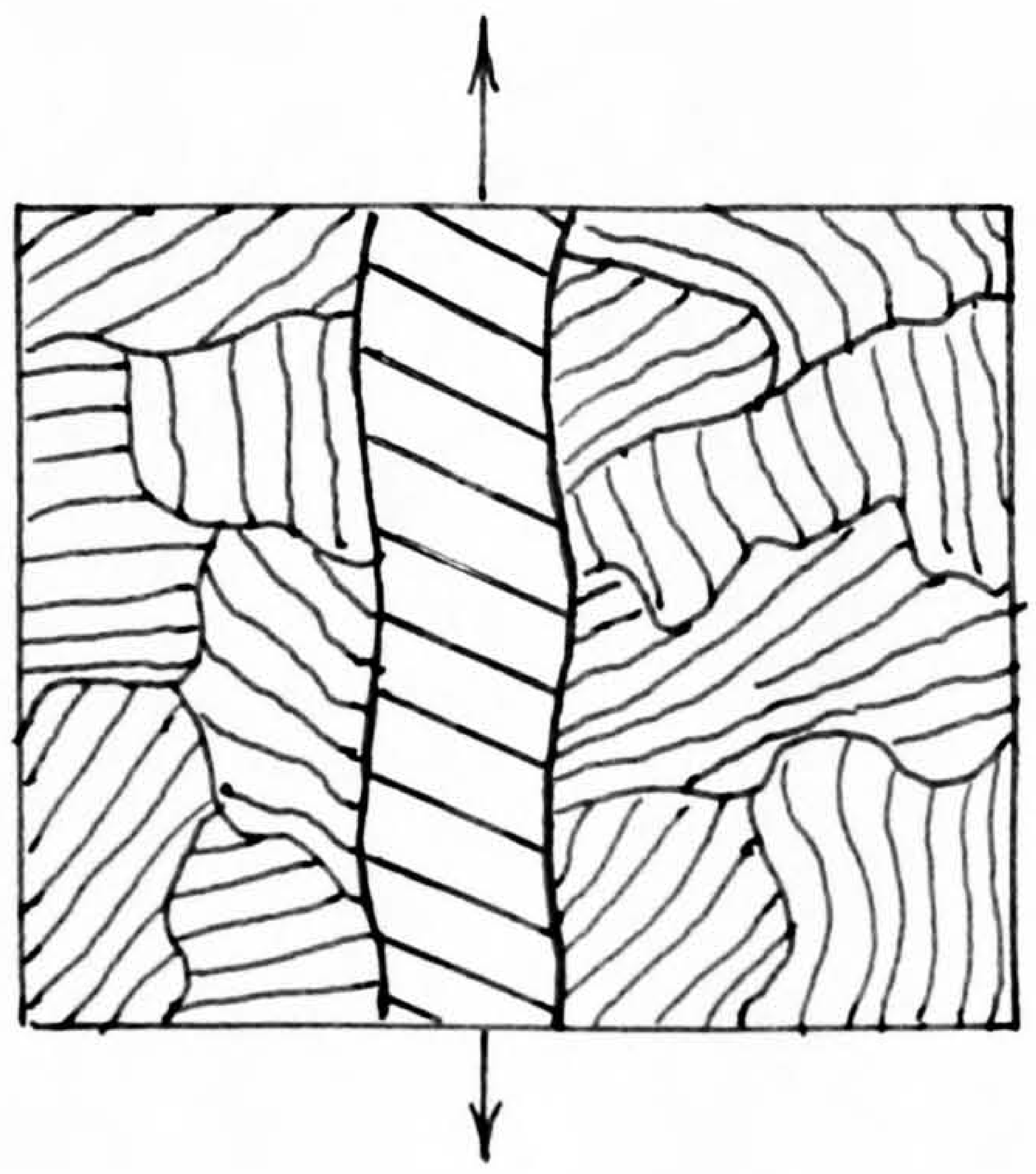


Fig. 5

along certain definite planes in the space lattice formed by the regular geometrical arrangements of atoms in a crystalline grain. Fine grained metals are usually stronger than coarse grained since the slip planes are shorter between interruptions.
As slip takes place the grain size directly controls the extent of this slip interference. The crystalline grains of metal are fragmented into thin plates and the surface of these plates dig into each other, causing increased resistance as slip proceeds in adjacent planes.

<u>Inertial Characteristics of Materials</u>

Newton's second law states that an unbalanced force causes a proportional rate of change of momentum that takes place in the direction in which the force is impressed.

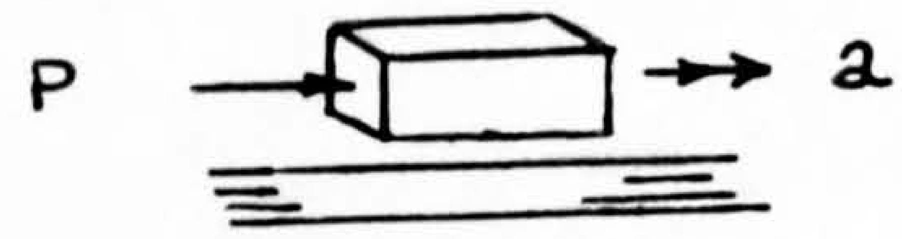


Fig. 6

The inertia theory is developed directly from Newton's second law and best illustrated by d'Alembert. He suggests that a fictitious force"F"be assumed to be added to the applied forces, a force equal in magnitude to that required for the acceleration (i.e., $W/g \times a$) but opposite in direction. This is then a "reversed effective force," and within it the body appears to be in equilibrium. $P - W/g \times a = 0$

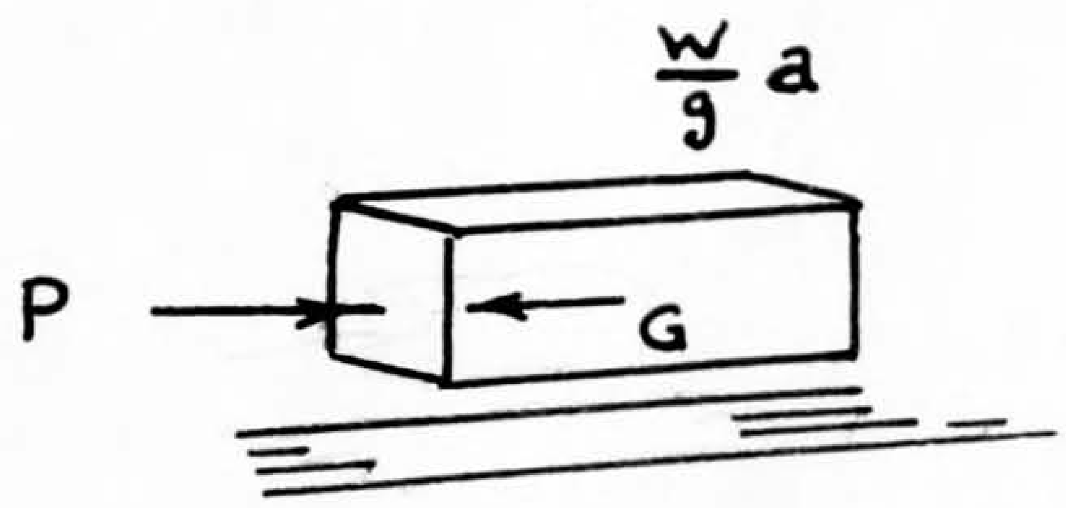


Fig. 7

Actually Newton's law states that for any number of real forces F acting on a body, the sum of their components "F_x" in any chosen direction "x" will produce an acceleration "a_x" in that direction. "F_x" = $W/g \times$ "a_x"

Therefore when each small particle of mass in the thin wall of the tube is deformed in a certain direction due to the initial load force, an acceleration of the tube mass in the same direction will exist. This will induce the inertia force or resistance to acceleration of the mass which will act opposite to the acceleration direction.

The acceleration of each small particle of mass is a function of the velocity at which the initial load is applied. Consequently the inertia force of

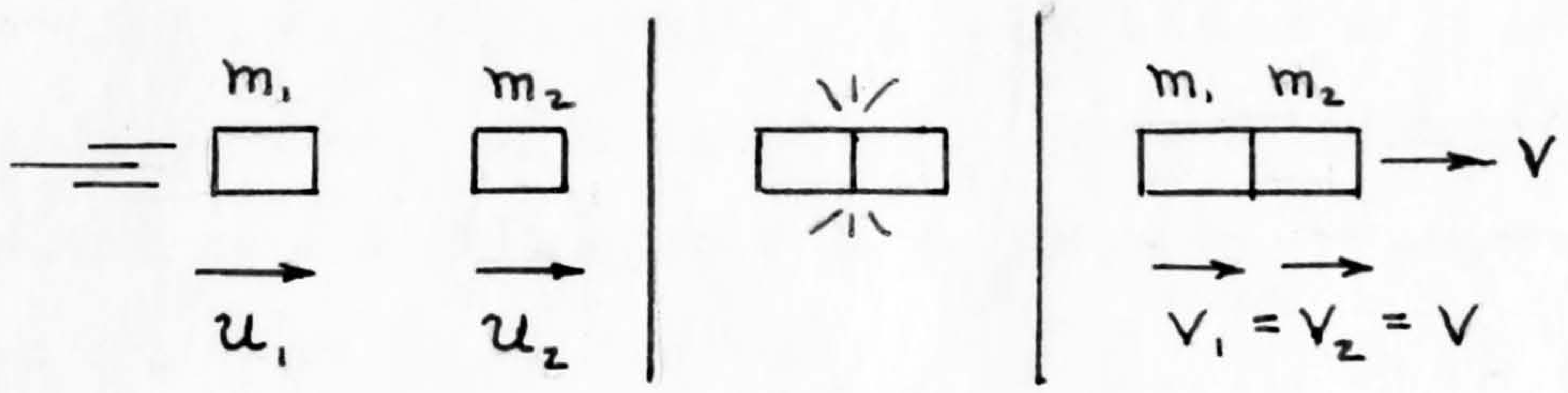


Fig. 8

each mass particle will be a function of the initial load velocity.

The increase in acceleration of the tube mass as the initial load velocity increases can be shown by impulse and momentum equations.

$$m_1 u_1 = \int F\,dt + m_1 v_1 \qquad m_2 u_2 + \int F\,dt = m_2 v_2$$

$$\int F\,dt = m_1 u_1 - m_1 v_1 \qquad u_2 = 0 \quad \therefore \int F\,dt = m_2 v_2$$

$$v_1 = v_2$$

$$m_1 u_1 - m_1 v = m_2 v$$

$$m_1 u_1 = m_1 v + m_2 v$$

$$m_1 u_1 = v\,(m_1 + m_2)$$

It can be seen from the above equation that as the initial load velocity "u_1" increases, the tube mass velocity "v" will increase. Thus as dv increases, the acceleration and inertial force will increase.

$$\int F\,dt = m_2 v \qquad F = m_2 \frac{dv}{dt} = m_2 a$$

CHAPTER V

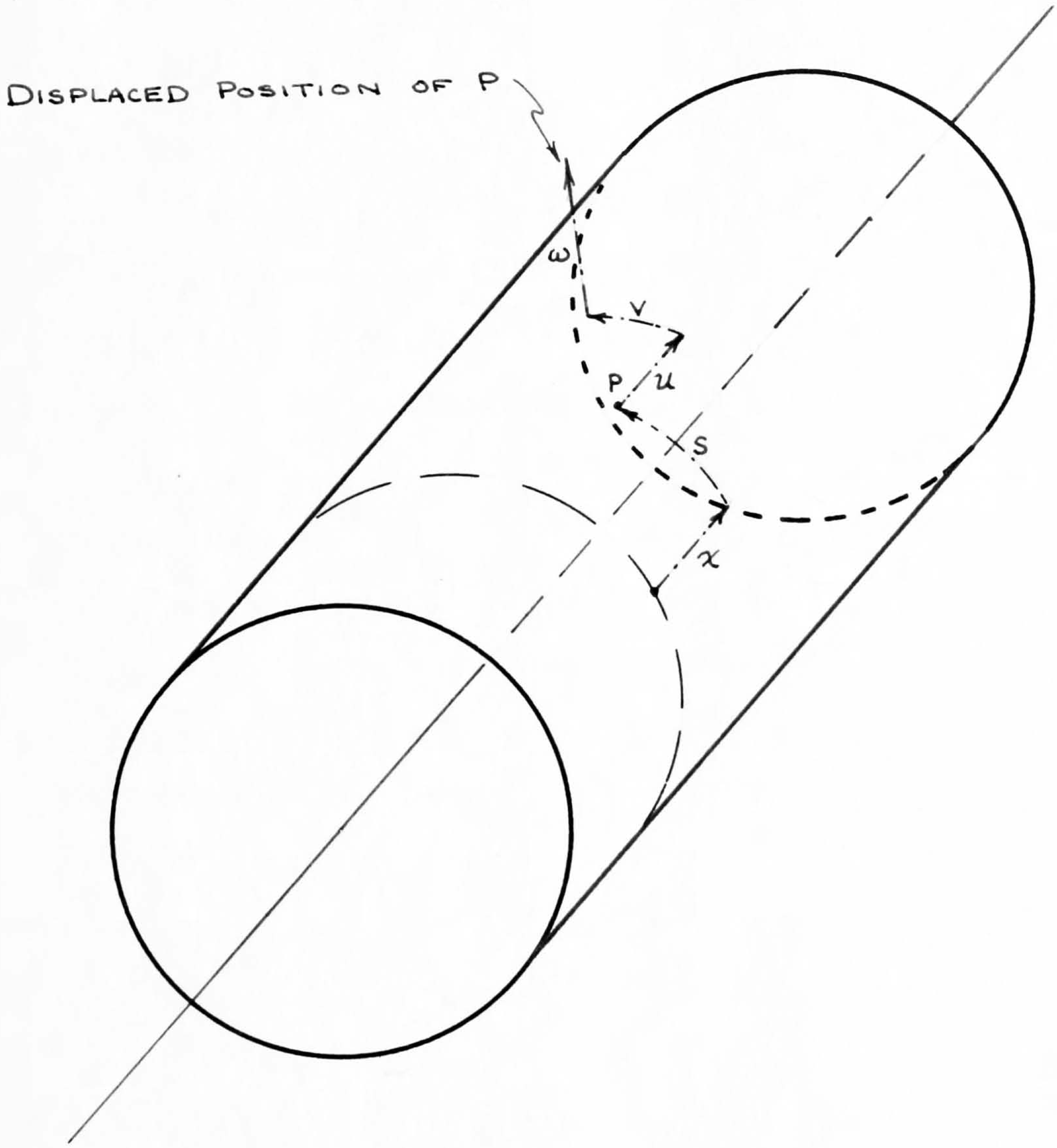
STRESS ANALYSIS

The equations of equilbirium of elements of the cylindrical wall of the tube have been obtained in a new and simplified form; consequently it will be necessary to give a derivation. Figure 9 shows the coordinates and the components of displacement of the middle surface of the wall during buckling. A circumferential coordinate "S" is used in preference to an angular one, because it results in simpler expressions and makes the connection between a curved plate and the limiting case of a flat plate more readily seen. To the order of approximation needed, it made no difference to L.H. Donnell whether the component of displacement "v" is considered to be measured circumferentially or tangentially.

---

loc. cit.

The equations of equilibrium for a flat plate are well known, but the corresponding equations for the case of a curved plate are by no means so clearly established. In the case of a curved plate, extension and flexure are, in general, interconnected even when the lateral deflections are of infinitesimal order. If no simplifications were made the conditions of equilibrium would be too complex to be of much practical



COORDINATES AND COMPONENTS OF DISPLACEMENT

Fig. 9

use. In the following discussion, an attempt was made by Mr. Donnell to obtain the greatest simplification possible under the conditions of the problem.

The usual assumptions are made, that the material is perfectly elastic, that the tube is exactly cylindrical, that the wall thickness is small compared to the radius, and that the deflections are small compared to the thickness. The usual assumption is also made that straight lines in the cylinder wall, perpindicular to the middle surface, remain straight and perpindicular to the middle surface; that is, the distortion due to transverse shear is neglected.

If lines perpindicular to the middle surface remain so during distortion then the displacement of all points in the cylinder wall can be found from the displacements of the middle surface u, v, and w. The equations of equilibrium can then be derived in terms of u, v, and w by considering: first, the purely geometrical relationship between these displacements and the strains in all parts of the wall; next, the relationship between all the strains and the stresses, given by Hooke's and Poisson's relations; and last, the relationship between all the stresses on an element of the wall, given by the laws of equilibrium.

The extensional and flexural strains in the middle surface are

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_s = \frac{\partial v}{\partial s} + \frac{w}{r} \qquad \varepsilon_{xs} = \frac{\partial u}{\partial s} + \frac{\partial v}{\partial x}$$

$$K_x = \frac{\partial^2 w}{\partial x^2} \qquad K_s = \frac{\partial^2 w}{\partial s^2} \qquad K_{xs} = \frac{\partial^2 w}{\partial x \partial s} \tag{1}$$

These expressions are the same as the expressions of the case of a flat plate, with the addition of w/r to the expression for ε_s. This term is due to the change in circumferential dimensions with change in the radius, which produces the strain:

$$\frac{r+w}{r} - 1 = \frac{w}{r}$$

The resultant forces and moments per unit length of wall section, obtained by summing up the stresses over the thickness, are taken as shown in Figure 10 The relation between these and the strains of the middle surface will be taken the same as in the case of a flat plate:

$$T_x = \frac{Et}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_s) \qquad T_s = \frac{Et}{1-\mu^2}(\varepsilon_s + \mu\varepsilon_x)$$

$$T_{xs} = T_{xs'} = \frac{Et}{2(1+\mu)}\varepsilon_{xs} \qquad G_x = \frac{Et^3}{12(1-\mu^2)}(K_x + \mu K_s)$$

$$G_s = \frac{Et^3}{12(1-\mu^2)}(K_s + \mu K_x) \qquad G_{xs} = G_{xs'} = \frac{Et^3}{12(1+\mu)}K_{xs}$$

In setting up the conditions for equilibrium of an element such as in Figure 10 , u, v, and w are the displacements occuring during buckling. Hence the above quantities T_x, G_x, etc., represent only the changes in the internal forces during buckling. The total internal forces at any instant are the internal forces present before buckling, plus these changes.

In this particular problem, the tube is subjected to torsion and, if the tube is perfectly cylindrical and uniform, the stress distribution and the distortion will be, before buckling begins, the same as assumed in elementary mechanics. There will be a shearing stress "S" on normal and longitudinal sections, which can be taken as uniform throughout the entire tube, since t/r is small. There will be a simple distortion in the circumferential direction, which leaves the tube still cylindrical. To obtain the total internal forces, the forces per unit length St must be added to those shown in Figure 10 and will be considered to be in the opposite sense to T_{xs} and T_{xs}'.

In setting up the conditions of equilibrium of the element, a consideration must be made of the changes in the angles of its faces due to distortion, as this will obviously affect the components of the forces in the different equilibrium equations. However, if the displacements are small this effect will be small, and its effect on T_x, G_x, etc., is of a second order of smallness compared to other items. But its effect on St may be of the same order of magnitude as these other items, because St is an order of magnitude larger than T_x, G_x, etc.; the latter forces are proportional to the buckling displacements and when these displacements are small, T_x, G_x, etc., must be small compared to St, which had a finite value when the buckling started.

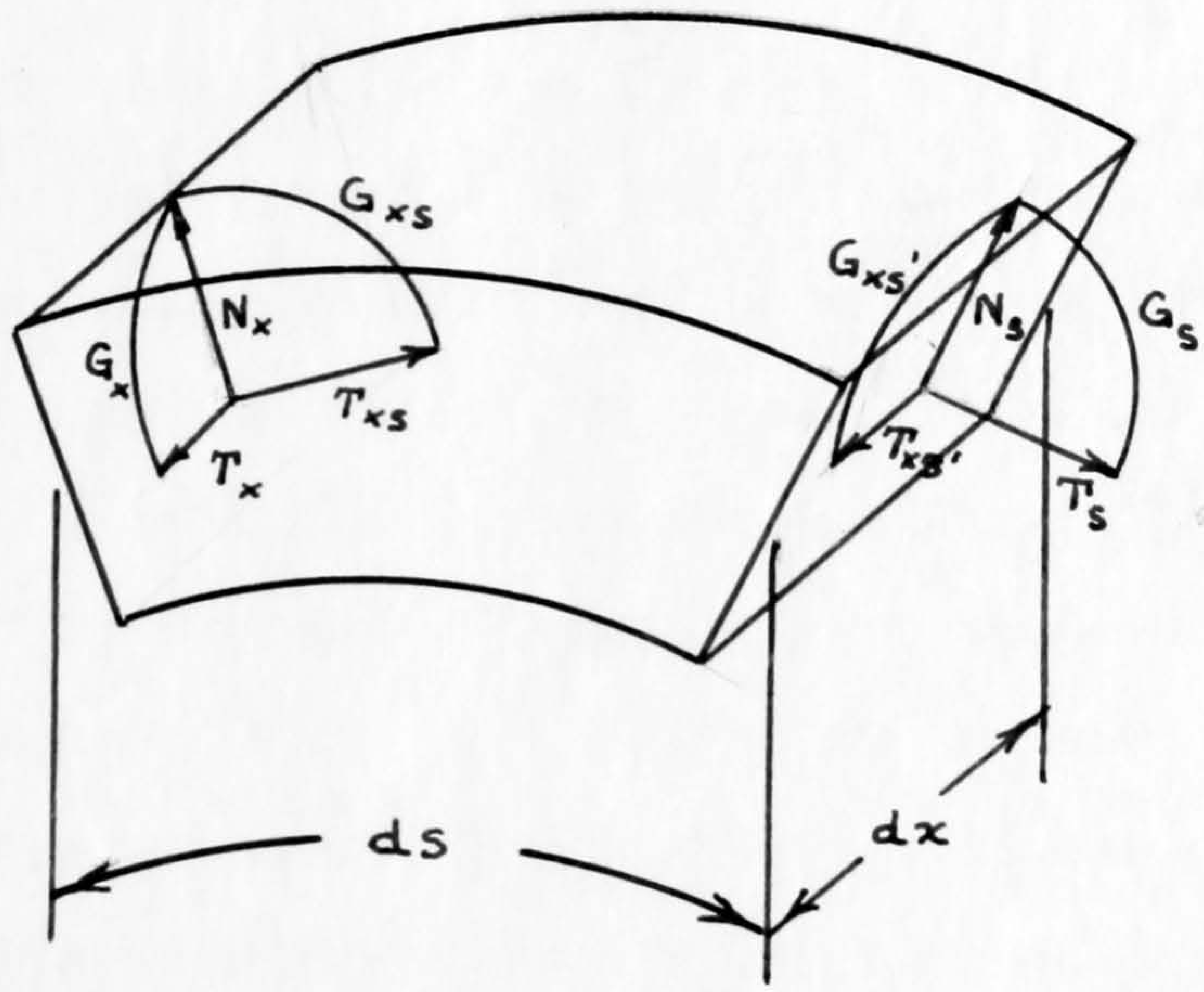


Fig. 10

The terms to be considered in the equations of equilibrium give, after simplification

$$\Sigma F_x = \frac{\partial T_x}{\partial x} + \frac{\partial T_{xs}'}{\partial s} = \rho \; v a$$

$$\Sigma F_s = \frac{\partial T_s}{\partial s} + \frac{\partial T_{xs}}{\partial x} = 0$$

$$\Sigma F_r = \frac{\partial N_x}{\partial x} + \frac{\partial N_s}{\partial s} + \frac{T_s}{r} + 2 s \tau \frac{\partial^2 \omega}{\partial x \partial s} = 0$$

$$\Sigma M_x = \frac{\partial G_s}{\partial s} + \frac{\partial G_{xs}}{\partial x} - N_s = 0$$

$$\Sigma M_s = \frac{\partial G_x}{\partial x} + \frac{\partial G_{xs}'}{\partial s} - N_x = 0$$

The first equation includes the inertia force which resists the distortion along the x axis or length of the tube. The inertia forces opposing distortion in the s and r directions will be ignored for reasons of simplification. The exclusion of these forces is justifiable since the inertia forces are negligible compared to the continuous positive buckling forces being applied in the same direction over the entire s and r distances. This is a result of the way in which the tubes are supported at the ends to enable the application of evenly distributed torque.

There is no use in writing the equation of moments about the radial direction, as it would merely state an original assumption- that $T_{xs} = T_{xs}'$. The term T_s/r in the third equation comes from the resultant of the force $T_s dx$ and the similar force on the opposite face of the element, due to the angle ds/r between them; this is the only term considered due to this angle, that is, due to the curvature of the element; all the other terms are the same as for a flat plate. The term 2 St is the only term considered due to the distortion of the element; this is the resultant of forces Stdx or Stds on all four sides of the element, due to the angle of twist between opposite sides, $\frac{\partial^2 \omega}{\partial x \partial s} dx$ OR $\frac{\partial^2 \omega}{\partial x \partial s} ds$. The rest of the terms are due to changes in T_x, G_x, etc., over the distances dx or ds, and to obvious moments due to N_x and N_s, the same as for a flat plate.

The above equations can be simplified and rewritten to approach a solution. The last two equations are solved simultaneously to eliminate N_x and N_s from the third, replacing T_x, G_x, etc., by their values in and then ε_s, K_x, etc., by their values in (1), three equations are obtained involving: derivatives of u, v, and w with respect to x and s, the unknown S, and the physical constants of the tube.

$$\Sigma F_x = \frac{\partial T_x}{\partial x} + \frac{\partial T_{xs'}}{\partial s} = \rho V a$$

ρ = DENSITY V = VOLUME OF MASS

a = ACCELERATION

$$T_x = \frac{Et}{1-\mu^2}\left(\frac{\partial u}{\partial x} + \mu \frac{\partial v}{\partial s} + \mu \frac{\omega}{r}\right)$$

$$\frac{\partial T_x}{\partial x} = \frac{EI}{1-\mu^2}\left(\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 v}{\partial s \partial x} + \frac{\mu}{r}\frac{\partial w}{\partial x}\right)$$

$$T_{xs}' = \frac{EI}{2(1+\mu)}\left(\frac{\partial u}{\partial s} + \frac{\partial v}{\partial x}\right)$$

$$\frac{\partial T_{xs}'}{\partial s} = \frac{EI}{2(1+\mu)}\left(\frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 v}{\partial x \partial s}\right)$$

$$\Sigma F_x : \frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 v}{\partial s \partial x} + \frac{\mu}{r}\frac{\partial w}{\partial x} + \frac{(1-\mu)}{2}\left[\frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 v}{\partial s \partial x}\right]$$

$$= \rho \, V a \left[\frac{1-\mu^2}{EI}\right]$$

$$= \frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 v}{\partial s \partial x} + \frac{\mu}{r}\frac{\partial w}{\partial x} + \frac{\partial^2 u}{2\partial s^2} + \frac{\partial^2 v}{2\partial s \partial x} - \frac{\mu \partial^2 u}{2\partial s^2} - \frac{\mu \partial^2 v}{2\partial x \partial s}$$

$$= \rho \, V a \left[\frac{1-\mu^2}{EI}\right]$$

$$\left[\frac{1-\mu^2}{EI}\right]\left[\frac{\partial^2 u}{\partial x^2} + \frac{(1-\mu)}{2}\frac{\partial^2 u}{\partial s^2} + \frac{(1+\mu)\partial^2 v}{2\partial s \partial x} + \frac{\mu}{r}\frac{\partial w}{\partial x}\right] = \rho \, V a$$

$$\Sigma F_s = \frac{\partial T_s}{\partial s} + \frac{\partial T_{xs}}{\partial x} = 0$$

$$T_s = \frac{EI}{1-\mu^2}\left[\frac{\partial v}{\partial s} + \frac{w}{r} + \mu\frac{\partial u}{\partial x}\right]$$

$$\frac{\partial T_s}{\partial s} = \frac{EI}{1-\mu^2}\left[\frac{\partial^2 v}{\partial s^2} + \frac{\partial w}{r\partial s} + \mu\frac{\partial^2 u}{\partial x \partial s}\right]$$

$$T_{xs} = \frac{Et}{2(1+\mu)}\left[\frac{\partial u}{\partial s} + \frac{\partial v}{\partial x}\right]$$

$$\frac{\partial T_{xs}}{\partial x} = \frac{Et}{2(1+\mu)}\left[\frac{\partial^2 u}{\partial s\,\partial x} + \frac{\partial^2 v}{\partial x^2}\right]$$

$$\Sigma F_s = \frac{Et}{1-\mu^2}\left[\frac{\partial^2 v}{\partial s^2} + \frac{\partial \omega}{r\,\partial s} + \mu\frac{\partial^2 u}{\partial x\,\partial s}\right] + \frac{Et}{2(1+\mu)}\left[\frac{\partial^2 u}{\partial s\,\partial x} + \frac{\partial^2 v}{\partial x^2}\right] = 0$$

$$= \frac{\partial^2 v}{\partial s^2} + \frac{\partial \omega}{r\,\partial s} + \mu\frac{\partial^2 u}{\partial x\,\partial s} + \frac{1-\mu}{2}\left[\frac{\partial^2 u}{\partial s\,\partial x} + \frac{\partial^2 v}{\partial x^2}\right] = 0$$

$$= \frac{\partial^2 v}{\partial s^2} + \frac{(1-\mu)\,\partial^2 v}{2\,\partial x^2} + \frac{(1+\mu)\,\partial^2 u}{2\,\partial x\,\partial s} + \frac{\partial \omega}{r\,\partial s} = 0$$

$$N_s = \frac{\partial G_s}{\partial s} + \frac{\partial G_{xs}}{\partial x}$$

$$\frac{\partial N_s}{\partial s} = \frac{\partial^2 G_s}{\partial s^2} + \frac{\partial^2 G_{xs}}{\partial x\,\partial s}$$

$$N_x = \frac{\partial G_x}{\partial x} + \frac{\partial G_{xs'}}{\partial s}$$

$$\frac{\partial N_x}{\partial x} = \frac{\partial^2 G_x}{\partial x^2} + \frac{\partial^2 G_{xs'}}{\partial s\,\partial x}$$

$$\frac{T_s}{r} = \frac{Et}{(1-\mu^2)r}\left[\frac{\partial u}{\partial s} + \frac{\omega}{r} + \mu\frac{\partial u}{\partial x}\right]$$

$$\Sigma F_r = \frac{\partial N_x}{\partial x} + \frac{\partial N_s}{\partial s} + \frac{T_s}{r} + 2 S t \frac{\partial^2 \omega}{\partial x \partial s} = 0$$

$$\Sigma F_r = \frac{\partial^2 G_s}{\partial s^2} + \frac{\partial^2 G_{xs}}{\partial x \partial s} + \frac{\partial^2 G_x}{\partial x^2} + \frac{\partial^2 G_{xs'}}{\partial s \partial x} +$$

$$\frac{Et}{(1-\mu^2) r}\left[\frac{\partial v}{\partial s} + \frac{\omega}{r} + \mu \frac{\partial u}{\partial x}\right] + 2 S t \frac{\partial^2 \omega}{\partial x \partial s} = 0$$

$$G_s = \frac{Et^3}{12(1-\mu^2)}\left[\frac{\partial^2 \omega}{\partial s^2} + \mu \frac{\partial^2 \omega}{\partial x^2}\right] \quad ; \quad \frac{Et^3}{12(1-\mu^2)} = C_1$$

$$\frac{\partial G_s}{\partial s} = C_1\left[\frac{\partial^3 \omega}{\partial s^3} + \mu \frac{\partial^3 \omega}{\partial x^2 \partial s}\right]$$

$$\frac{\partial^2 G_s}{\partial s^2} = C_1\left[\frac{\partial^4 \omega}{\partial s^4} + \mu \frac{\partial^4 \omega}{\partial x^2 \partial s^2}\right]$$

$$G_x = C_1\left[\frac{\partial^2 \omega}{\partial x^2} + \mu \frac{\partial^2 \omega}{\partial s^2}\right]$$

$$\frac{\partial G_x}{\partial x} = C_1\left[\frac{\partial^3 \omega}{\partial x^3} + \mu \frac{\partial^3 \omega}{\partial s^2 \partial x}\right]$$

$$\frac{\partial^2 G_x}{\partial x^2} = C_1\left[\frac{\partial^4 \omega}{\partial x^4} + \mu \frac{\partial^4 \omega}{\partial s^2 \partial x^2}\right]$$

$$G_{xs} = G_{xs'}$$

$$G_{xs} = \frac{E\mathcal{I}^3}{12(1+\mu)}\left[\frac{\partial^2 \omega}{\partial x \partial s}\right]$$

$$\frac{\partial G_{xs}}{\partial x} = \frac{E\mathcal{I}^3}{12(1+\mu)}\left[\frac{\partial^3 \omega}{\partial x^2 \partial s}\right]$$

$$\frac{\partial^2 G_{xs}}{\partial x \partial s} = \frac{E\mathcal{I}^3}{12(1+\mu)}\left[\frac{\partial^4 \omega}{\partial x^2 \partial s^2}\right]$$

$$\Sigma F_r = \frac{E\mathcal{I}^3}{12(1-\mu^2)}\left[\frac{\partial^4 \omega}{\partial s^4} + \mu\frac{\partial^4 \omega}{\partial s^2 \partial x^2} + \frac{\partial^4 \omega}{\partial x^4} + \mu\frac{\partial^4 \omega}{\partial s^2 \partial x^2}\right] +$$

$$\frac{2E\mathcal{I}^3}{12(1+\mu)}\left[\frac{\partial^4 \omega}{\partial x^2 \partial s^2}\right] + \frac{E\mathcal{I}}{(1-\mu^2)r}\left[\frac{\partial v}{\partial s} + \frac{\omega}{r} + \mu\frac{\partial u}{\partial x}\right] +$$

$$2S\mathcal{I}\frac{\partial^2 \omega}{\partial x \partial s} = 0$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial s^2} \qquad \nabla^4 = \frac{\partial^4}{\partial x^4} + \frac{2\partial^4}{\partial x^2 \partial s^2} + \frac{\partial^4}{\partial s^4}$$

MULTIPLY ΣF_r BY $\frac{1-\mu^2}{E\mathcal{I}}$

$$\Sigma F_r = \frac{\mathcal{I}^2}{12}\left[\frac{\partial^4 \omega}{\partial s^4} + \frac{2\mu\partial^4 \omega}{\partial x^2 \partial s^2} + \frac{\partial^4 \omega}{\partial x^4}\right] + \frac{2\mathcal{I}^2(1-\mu)}{12}\left[\frac{\partial^4 \omega}{\partial x^2 \partial s^2}\right] +$$

$$\frac{1}{r}\left[\frac{\partial v}{\partial s} + \frac{\omega}{r} + \mu\frac{\partial u}{\partial x}\right] + \frac{(1-\mu^2)}{E} 2S \frac{\partial^2 \omega}{\partial x \partial s} = 0$$

$$= \frac{\mathcal{I}^2}{12}\left[\frac{\partial^4 \omega}{\partial s^4} + \frac{2\partial^4 \omega}{\partial x^2 \partial s^2} + \frac{\partial^4 \omega}{\partial x^4}\right] + \frac{1}{r}\left[\frac{\partial v}{\partial s} + \frac{\omega}{r} + \mu\frac{\partial u}{\partial x}\right] +$$

$$\frac{(1-\mu^2)}{E} 2S \frac{\partial^2 \omega}{\partial x \partial s} = 0$$

$$\Sigma F_r = \frac{t^2}{12} \nabla^4 \omega + \frac{1}{r}\left[\frac{\partial v}{\partial s} + \mu \frac{\partial u}{\partial x} + \frac{\omega}{r}\right] + \frac{2(1-\mu^2)}{E} S \frac{\partial^2 \omega}{\partial x \partial s} = 0$$

$$\Sigma F_x = \left[\frac{1-\mu^2}{E t}\right]\left[\frac{\partial^2 u}{\partial x^2} + \frac{(1-\mu)\partial^2 u}{2\partial s^2} + \frac{(1+\mu)\partial^2 v}{2\partial s \partial x} + \frac{\mu \partial \omega}{r \partial x}\right] = \rho \quad V a$$

$$\Sigma F_s = \frac{\partial^2 v}{\partial s^2} + \frac{(1-\mu)\partial^2 v}{2\partial x^2} + \frac{(1+\mu)\partial^2 u}{2\partial x \partial s} + \frac{\partial \omega}{r \partial s} = 0$$

$$\Sigma F_r = \frac{t^2 \Delta^4 \omega}{12} + \frac{1}{r}\left[\frac{\partial v}{\partial s} + \mu \frac{\partial u}{\partial x} + \frac{\omega}{r}\right] + \frac{2(1-\mu^2)}{E} S \frac{\partial^2 \omega}{\partial x \partial s} = 0$$

The force equations above could be further simplified and a solution found by seperating the variables of velocity and deformation x , integrating, and including the physical constants. However, this is beyond the scope of this thesis.

In order to have a better understanding of the characteristic deformation of a thin-walled tube, it is necessary to inspect and analyze the distortion along the length of a tube, which accompanies a torsional inelastic failure.

In torsional inelasticity failure, a number of circumferential waves "n" will appear down the length of the tube. The case n=1 would give a distortion in which cross sections remain circular but are displaced, the displacement spiralling around the center line, so that the shape of the tube would become something like that of a corkscrew.

In this case the elementary theory of bending of a tube applies. Figure 11 shows a tube undergoing this type of distortion, under the action of a twisting moment M_t , the center line being bent to a spiral and having the constant angle θ with the axis of the spiral. If the couple M_t acts about the axis of the spiral, all parts of the tube will be subjected to the bending moment $M_t \sin \theta$. At the same time it can be shown that all parts of the tube are bent to a curvature $\sin^2 \theta/R$ (where R is the radius of the spiral). This curvature is in the same plane as the bending moment $M_t \sin \theta$. Hence all parts of the tube will be in equilibrium if $M_t \sin \theta = E\,I \sin^2 \theta/R$. If the end

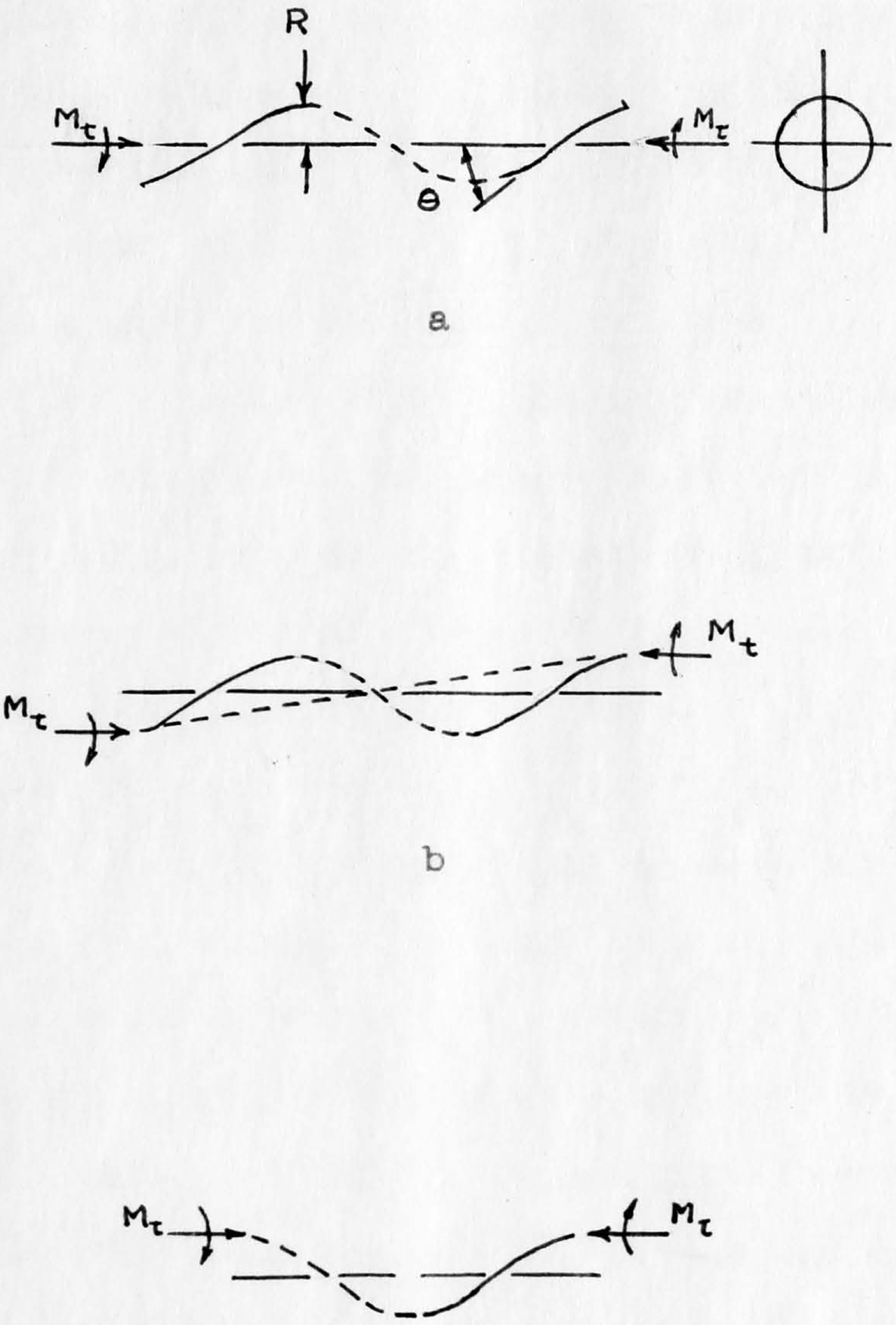


Fig. 11

conditions are such that $\sin \Theta/R$ can have only one particular value, as in the case discussed in the next paragraph, then this formula determines a value of M_t at which the tube can buckle in the shape given.

It was assumed above that the couple M_t is applied at the axis of the spiral. In a practical case it would be applied at the end of the tube, as shown in Figure 11b . As the couple has only been moved parallel to itself this is statically equivalent to the case of Figure 11a, and the above reasoning still applies. But now the couples at the ends of the tube are not about the line joining the two ends. In order to fulfill a requirement that the end couples be about this line, the spiral form of the tube must consist of an even number of full turns. The condition for this is that $\sin \Theta/R = m\, 2\pi/L$ where m is an integer. The loading conditions correspond to those assumed by Greenhill but the loading applied in actual experiments can not be called pure twisting moment, as the applied couple is not about the axis of the tube at the end.

A.G. Greenhill, Strength of Shafting when Exposed both to Torsion and End Thrust, Proc. Inst. of Mech. Eng., London, 1883, P.182.

In this analysis of buckling, lateral deflection merely amounts to a change of a component of the twisting moment into bending moment. The resulting deflections could never be as great as the bending deflections which would occur if the whole twisting moment were to be applied as a bending moment. In the case of a long piece of rubber tubing, enormous angles of twist can be obtained. This deformation is not especially apparent, as it leaves the tube cylindrical as before; if, now, some of this twisting deformation suddenly goes into bending deformation, the resulting deformation is very spectacular, even if the angles of bending are only a small part of the previous angles of twist.

Donnell, op.cit. p. 113

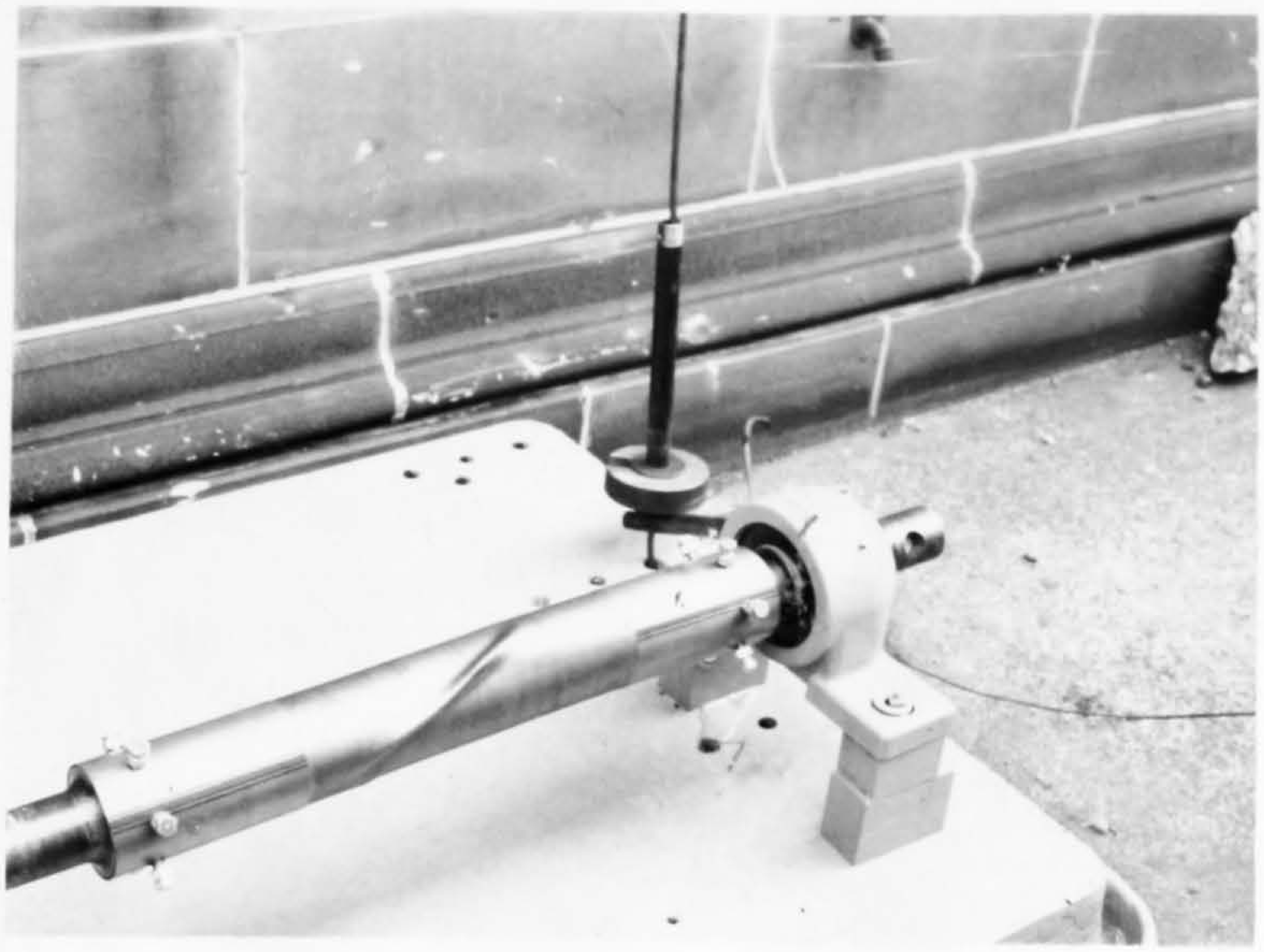
CHAPTER VI

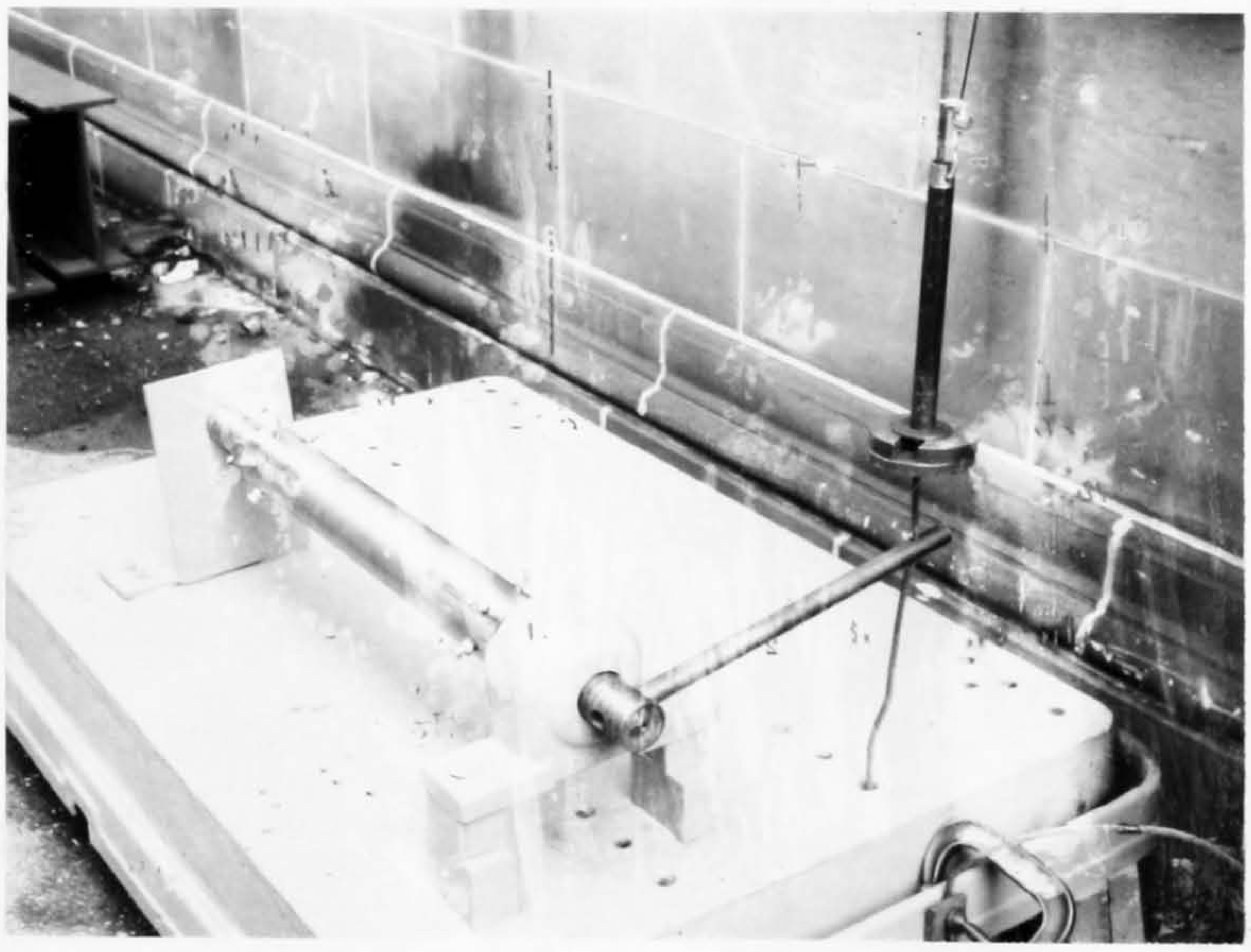
TEST APPARATUS

In order to determine a correlation between the theoretical analysis and the experimental results, the next step in this investigation was to subject some tubes to dynamic torsion. This chapter describes the test samples and the apparatus used to torsion test the samples.

1. The tube specimens used were 2 ft. long, cold rolled,welded, round steel tubes with a $3\frac{1}{2}$" O.D. and 16 gage wall thickness. The tubes were turned down to 0.010" wall thickness along a centered length of 12" to permit better support and locate the failure. The steel was from C1015 standard stock. Six tubes were used in the investigation,and six more will be used in subsequent tests to varify the results of this thesis.

2. The adapters to apply torque to the tubes were two step-down round steel bars with six tapped holes in each O.D. as shown in the photograph. The large diameter was machined to the inside dimensions of the tube, and then the adapter was placed inside the tube securely fastened by six screws on each end. It was thought that pre-stressing the holding screws gave a





constant friction force between the adapter and the tube. This maintained a uniform torque completely around the tube.

3. The torsion testing machine used was a lever arm apparatus fabricated in the U. of D. machine shop. It consisted of the tube adapters simply supported by a pillow block bearing at one end and a stationary plate at the other end. The stationary plate held one end of the tube fixed while the bearing permitted the other end of the tube to rotate as a torque was transferred to the tube. The torque was obtained by weights placed on a 18" lever arm attached to the rotating tube adapter as shown in the pictures.

4. A dynamic torsion was obtained by dropping weights down a guide line from the roof of the Engineering building. This allowed for control of the dropping weights, which were 10 and 15 pounds.

A stress analysis was used in the design of the test apparatus in order to insure its satisfactory performance.

CHAPTER VII

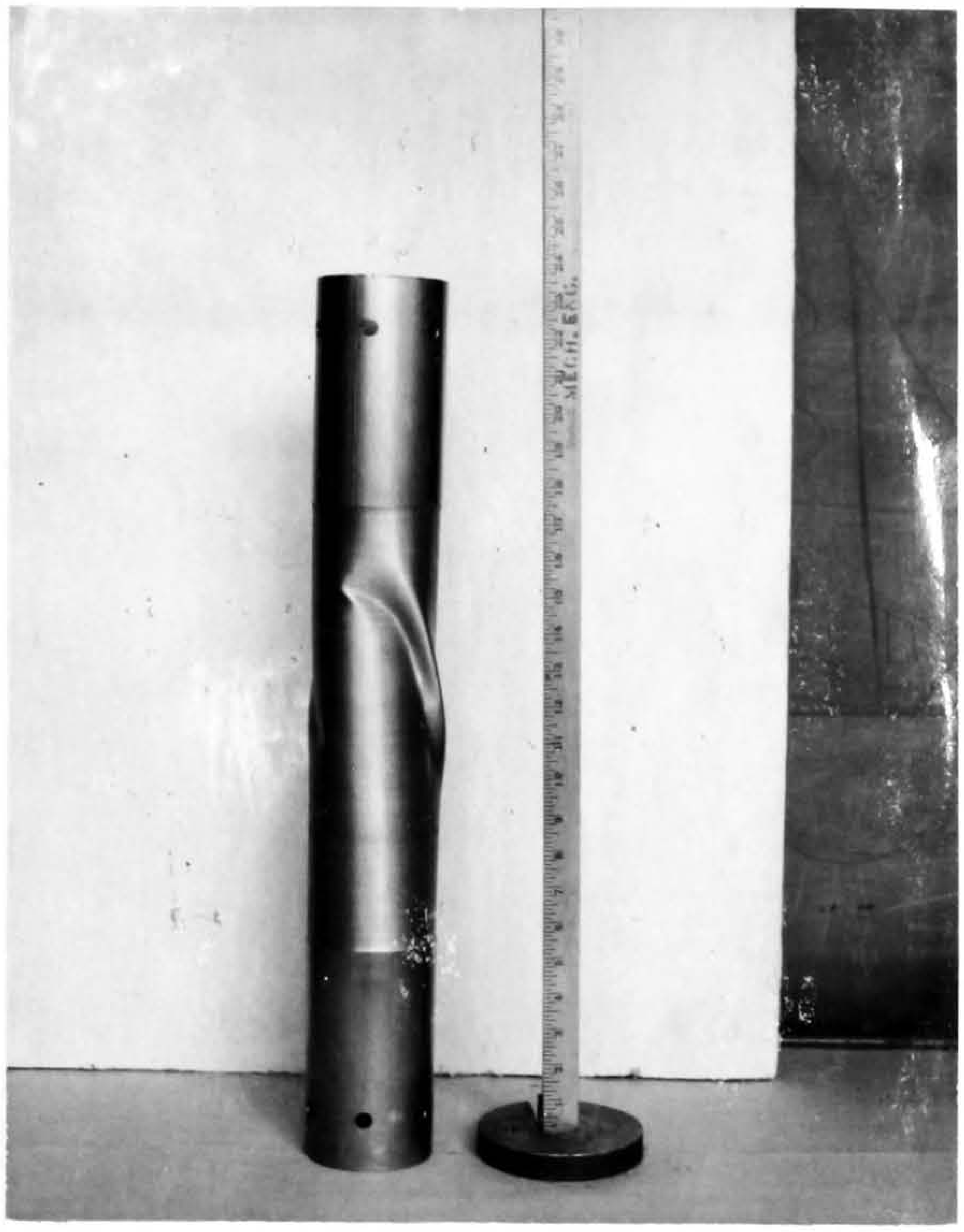
PROCEDURE

The procedure for testing consisted of a means that would twist the tubes at different velocities and energy levels. To accomplish this various weights were dropped from different heights onto the lever arm apparatus mentioned in the last chapter.

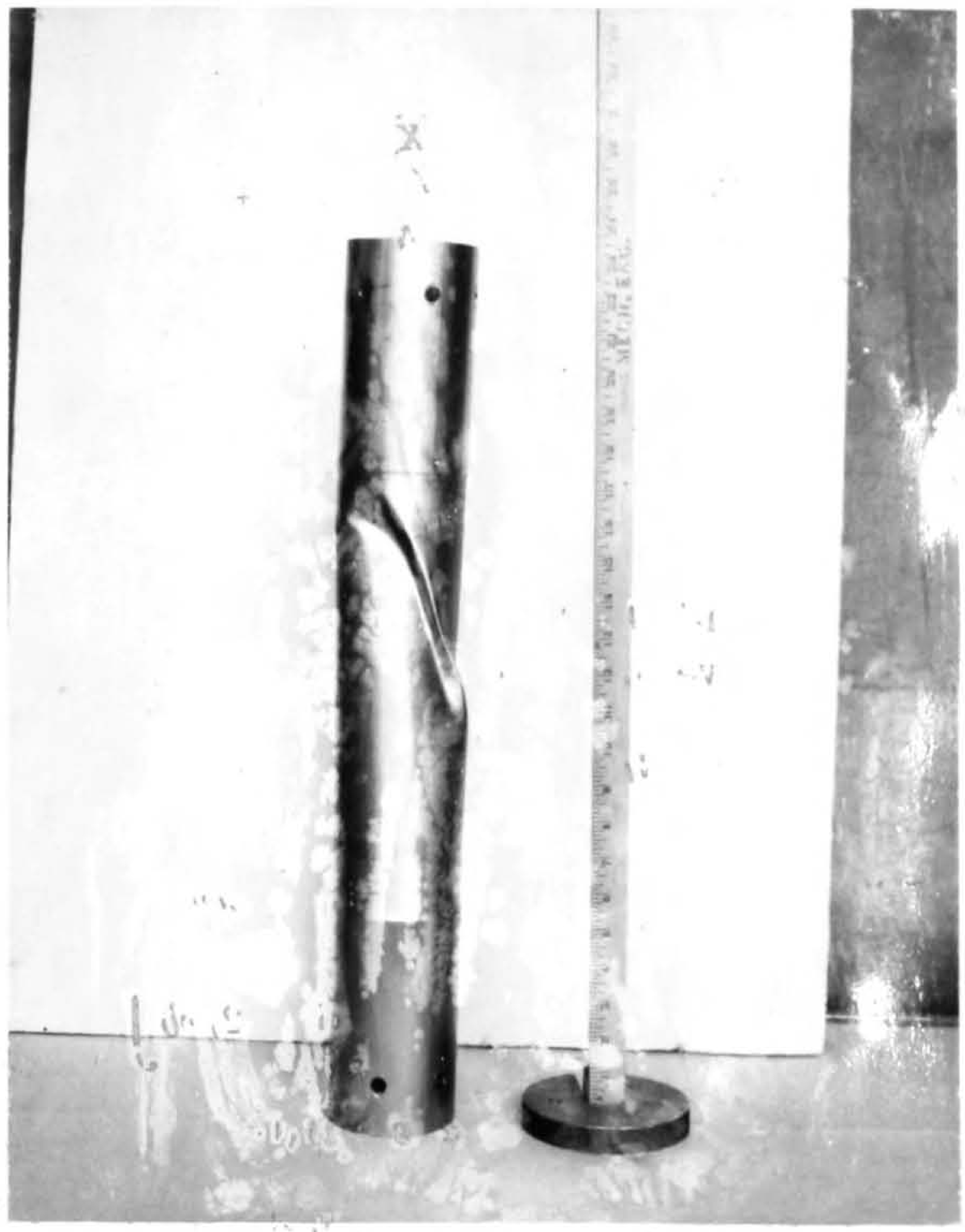
Six tubes were tested, a first tube being tested statically to determine the necessary torque and characteristic location of deformation. A second tube was used when high speed pictures were taken of it as the tube buckled.

The other four tubes were used in the experimental dynamic torsion tests. Tube 1 was dynamically torqued when a 10 lb. weight was dropped from 20 feet high onto the lever arm. Tube 2 was tested by dropping a 10 lb. weight from a height of 30 feet. Tube 3 was twisted dynamically when a 15 lb. weight was dropped from 20 feet onto the lever arm. Tube 4 was tested by dropping a 15 lb. weight from a height of 30 feet.

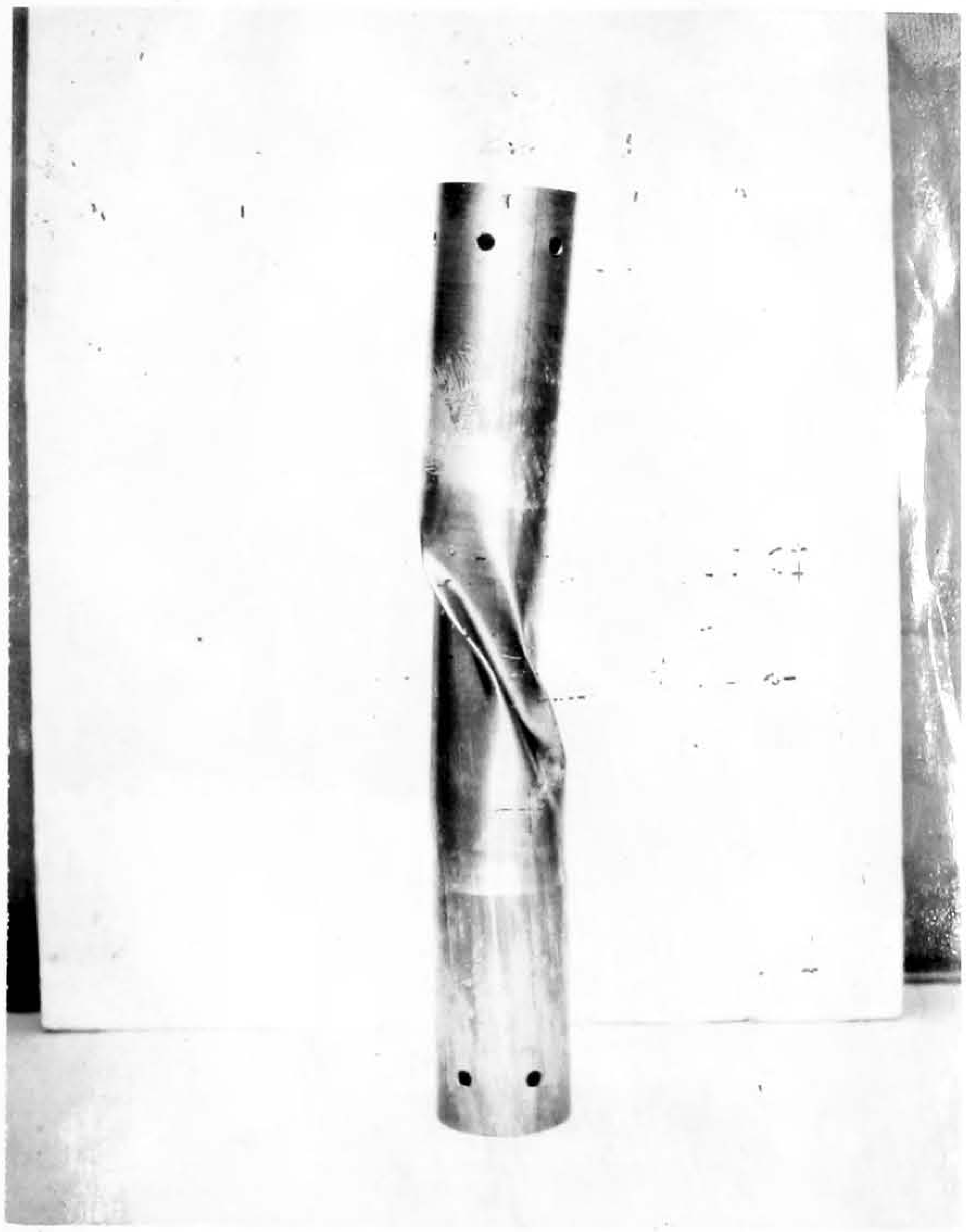
The desired velocities were obtained by calculating the height needed to reach a free fall velocity equal to the desired velocity. The weights were then dropped from this height. The buckled tubes were then inspected, and their deformation locations recorded along with the corresponding torques and velocities.



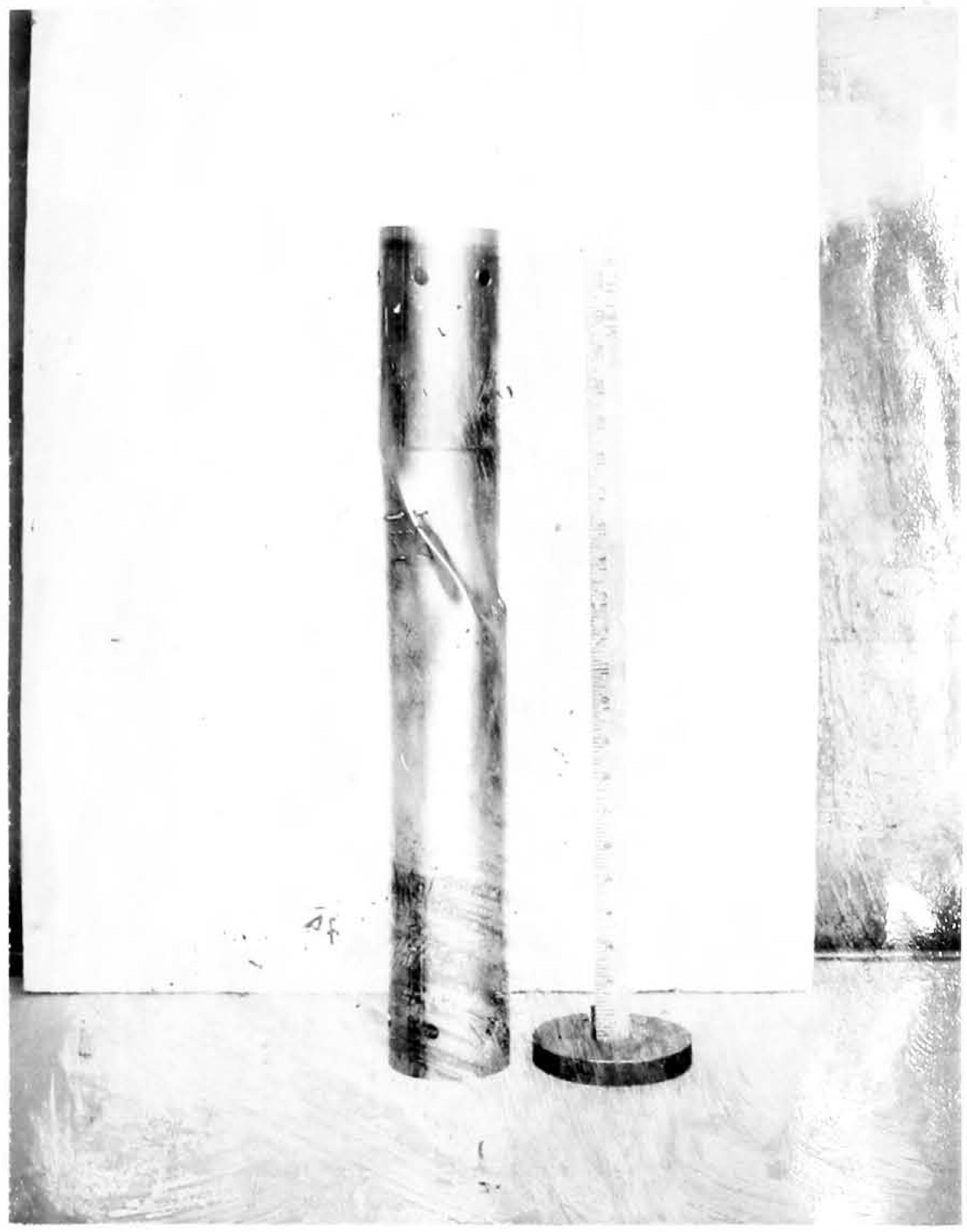
Tube 1



Tube 2



Tube 3



Tube 4

CHAPTER VIII

EXPERIMENTAL AND CALCULATED DATA

	Load	Velocity
Tube (Static)	300 lb.	0
Tube 1	10 lb.	36 ft/sec
Tube 2	10 lb.	44 ft/sec
Tube 3	15 lb.	36 ft/sec
Tube 4	15 lb.	44 ft/sec

Characteristic Deformation

	Ave. distance from end load was applied	Ave. depth of radial distortion
Tube (Static)	12 inches	--
Tube 1	10.5 inches	1 inch
Tube 2	9 inches	3/4 inch
Tube 3	11 inches	$1\frac{1}{2}$ inch
Tube 4	8.75inches	3/4 inch

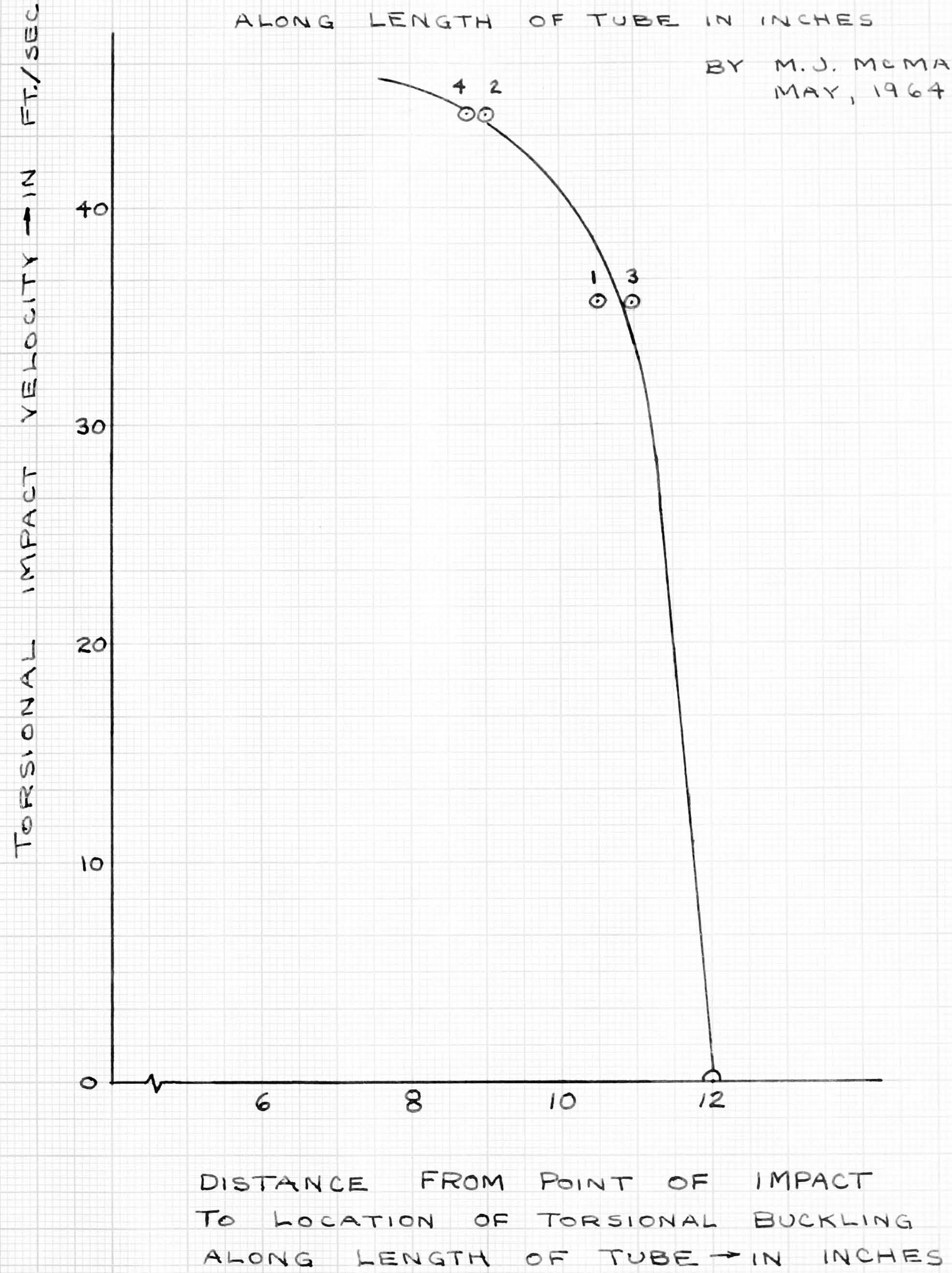
Sample Calculations

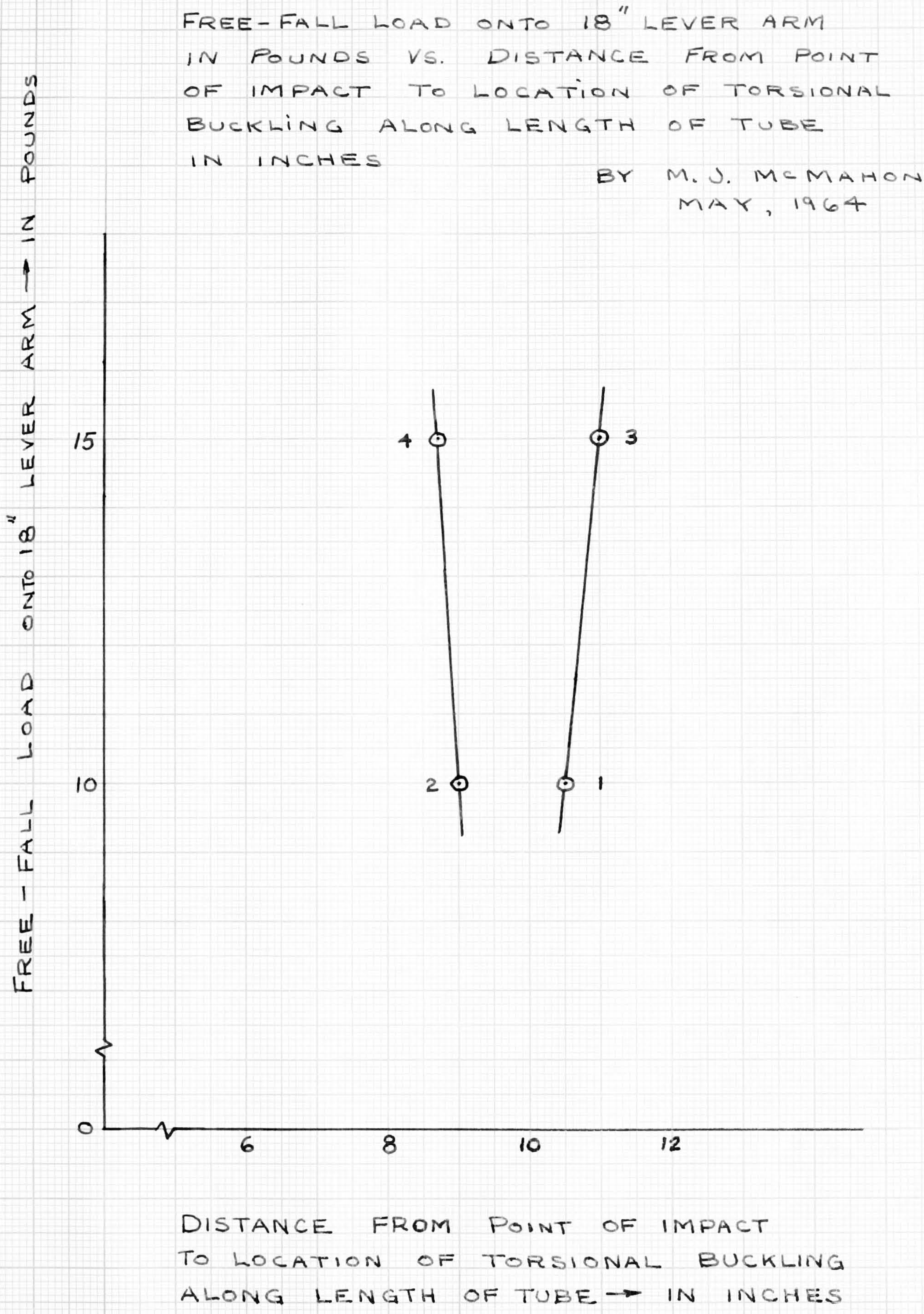
$$V = 2gh$$

Height	Velocity
20 ft.	36 ft/sec
30 ft.	44 ft/sec

TORSIONAL IMPACT VELOCITY IN FT./SEC. VS. DISTANCE FROM POINT OF IMPACT TO LOCATION OF TORSIONAL BUCKLING ALONG LENGTH OF TUBE IN INCHES

BY M. J. MCMAHON
MAY, 1964




FREE-FALL LOAD ONTO 18" LEVER ARM IN POUNDS VS. DISTANCE FROM POINT OF IMPACT TO LOCATION OF TORSIONAL BUCKLING ALONG LENGTH OF TUBE IN INCHES
BY M. J. McMAHON
MAY, 1964
FREE-FALL LOAD ONTO 18" LEVER ARM → IN POUNDS
15
10
0
4
3
2
1
6
8
10
12
DISTANCE FROM POINT OF IMPACT TO LOCATION OF TORSIONAL BUCKLING ALONG LENGTH OF TUBE → IN INCHES

CHAPTER IX

RESULTS AND INTERPRETATION OF DATA

The results of this experimental investigation prove that the theory on the characteristic deformation in a tube buckling from dynamic torsion is correct.

The twisting deformation is located at a distance along the tube further from the point of torsional impact as the torsion velocity decreases. The location of this distortion along the length during buckling is a direct dependent function of the velocity and independent of the torsional load applied. This result is concluded from inspection of the destroyed tubes.

The graphs of distortion vs. load and distortion vs. velocity indicate that the location of deformation is an independent function of the torque and a dependent function of the velocity. The distortion vs. velocity curve indicates that as the velocity increases the deformation moves down the length of the tube. The deformation vs. torsion curve shows two straight vertical lines independent of each other. This indicates the the independence of amount of torsion to the deformation location on the tube.

Tubes 1 and 3 were destroyed by dropping different weights from the same height. This would mean that the tubes during buckling would absorb different amounts of energy,but the loads would be applied at the same velocity. The characteristic deformation of tubes 1 and 3 shows that for both tubes the major portion of the circumferential wave was located the same distance from the end where the load was applied. This distance was approximately 10.75 inches. However,the radial deformation was not the same for tubes 1 and 3.

Tube 3 which had a greater amount of energy to absorb due to the larger load applied to it, had greater radial distortion in the circumferential wave. The twist in tube 3 was 1/2 inch deeper than tube 1 indicating that the extra 5 lb. load energy was dissipated in radial distortion after the buckling distortion was accomplished.

Torsional buckling produced circumferential waves in the tubes. Tube 1 had two waves in it, one on each side of the tube. This showed that a more uniform torque was applied to it. The other tubes contained only one circumferential wave from a less uniform torque application. The uniformity of the torque application by the apparatus was limited since the lever arm tends to cause bending.

It was observed that the weld in the tube did not have an appreciable effect on the buckling. The weld seemed to be equally as strong as the rest of the tube since two tubes buckled along the weld and two tubes did not.

CHAPTER X

CONCLUSIONS AND RECOMMENDATIONS

It has been shown that the distance from the point of torsional impact to the location of buckling is dependent on the speed at which the torsion is applied. As the torsional speed is increased, the distance will decrease. This phenomenon is best explained by the inertia force theory. As the velocity of impact increases, the retarding inertia force increases and only permits the buckling force to distort the tube close to the point of loading.

A better comparison of the relationship between impact velocity and location of deformation could be made by maintaining a constant energy load. A constant level of energy should be applied to the tube during torsional buckling. By properly decreasing the amount of weight dropped as the height is increased, a constant buckling energy will be absorbed during tube distortion.

Tubes of a more uniform thickness and homogeneous metal without a weld would be more ideal; although tubes of this quality would not be as realistic as those tubes found in general use. It should be recommended that tubes with a smaller wall thickness should be used to reduce the size and inefficiency of the test apparatus by reducing the inertia of the rotating parts.

BIBLIOGRAPHY

F.R.E. Crossley- Dynamics in Machines
Ronald Press Co.
New York, 1954

L.H. Donnell---- Stability of Thin-Walled Tubes under Torsion, National Advisory Committee for Aeronautics, Report No. 479, 1934

Glenn Murphy---- Mechanics of Materials
Irwin Farnham Publishing Co.
Chicago, Illinois 1948

Den Hartog------ Advanced Strength of Materials
McGraw Hill Book Co.
New York, 1952

Arthur Murphy--- Torsional Buckling Characteristics of Thin-Walled Slit Steel Tubing
Univ. of Detroit Thesis, 1960

Rinehart and Pearson- Behavior of Metals under Impulsive Loads, American Society for Metals, Haddon Craftsmen Inc., Scranton, Penn., 1954

S. Timoshenko--- Strength of Materials
D. Van Nostrand Corp. New York, 1930

Theory of Elastic Stability
McGraw Hill Book Co.
New York, 1936

Theory of Plates and Shells
McGraw Hill Book Co.
New York, 1940

S. Timoshenko and D.H. Young- Elements of Strength of Materials, D. Van Nostrand Corp., New York, 1962

---------------- Distribution of Stresses in a Structure under Dynamic Loading, Experimental Stress Analysis, Vol.IV, No.2.